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First Semester M.Tech. Degree Examination, Dec.2014/Jan.2015
Advanced Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Construct QR decomposition for the matrix.

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$ **(08 Marks)**

b. Find the pseudo inverse of matrix A and verify all the properties of pseudo inverse. **(12 Marks)**

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 0 & 2 \end{bmatrix}$$

2 a. Find singe value decomposition of matrix $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$. **(10 Marks)**

b. Find a least-squares solution of Ax = b of the following equations.
$x_3 + 2x_4 = 1$
$x_1 + 2x_2 + 2x_3 + 3x_4 = 2$
$x_1 + 3x_2 + 2x_3 = 4$. **(10 Marks)**

3 a. Find the extremum of functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y^{1^2}}}{y} dx$. **(06 Marks)**

b. Find the extremum of functional
$v[y(x)] = \int_0^1 \left(xy + y^2 - 2y^2 y^1\right) dx$ $y(0) = 1$ $y(1) = 2$. **(06 Marks)**

c. Show that the extremum of functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y^{1^2}}}{x} dx$ is a family of circles and also determine centre and radius of circle which is passing through (1, 0) and (3, 5). **(08 Marks)**

4 a. Find the extremals of the functional
$v[y(x)] = \int_{x_0}^{x} \left[(y^{11})^2 - 2(y^1)^2 + y^2 - 2y\sin x\right] dx$. **(08 Marks)**

b. Find a function y(x) for which $\int_0^1 (x^2 + y^{1^2}) dx$ is stationary given that $\int_0^1 y^2 dx = 2$; $y(0) = y(1) = 0$. **(12 Marks)**

5 a. An infinitely long string having one end at x = 0 is initially at rest on the x – axis. The end x = 0 undergoes a periodic transverse displacement described by $A_0 \text{Sin } wt$, $t > 0$. Find the displacement of any point on the string at any time t. Solve by Laplace transform method. **(10 Marks)**

b. Solve the heat conduction problem described by PDE : $K\frac{\partial^2 u}{0x^2} = \frac{\partial u}{\partial t}$ $\quad 0 < x < \infty$ t 70 with condition $u(0, t) = u0 \quad t \geq 0$; $u(x, 0) = 0 \quad 0 < x < \infty$; u and $\frac{\partial u}{\partial x}$ both tend to zero as $x \to \infty$. **(10 Marks)**

6 a. Solve the following potential problem in the semi-infinite strip described by $u_{xx} + u_{yy} = 0$, $0 < x < \infty$, and $0 < y < a$ subject to $u(x, 0) = f(x)$, $x(x, a) = 0$, $u(x, y) = 0$, $0 < y < a$, $0 < x < \infty$ and $\frac{\partial u}{\partial x}$ tends to zero as $x \to \infty$. **(10 Marks)**

b. One dimensional infinite solid $-\infty < x < \infty$, is initially at temperature F(x). For times $t > 0$, heat is generated with in solid at a rate of g (x, t) units. Determine the temperature in the solid for $t > 0$. **(10 Marks)**

7 a. Use simplex method to solve the following equation

Maximize $Z = 5x_1 + 2x_2$

Subject to $6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$

with all variable non negative. **(10 Marks)**

b. Determine the symmetric dual of the program. Show that both primal and dual program have the same optimal value for Z.

Maximize $Z = 2x_1 + x_2$

Subject to $x_1 + 5x_2 \leq 10$

$x_1 + 3x_2 \leq 6$

$2x_1 + 2x_2 \leq 8$

with all variables non negative. **(10 Marks)**

8 a. Solve the following program by use of Lagrange-multipliers:

Minimize $Z = x_1 + x_2 + x_3$

Subject to $x_1^2 + x_2 = 3$

$x_1 + 3x_2 + 2x_3 = 7$. **(10 Marks)**

b. Solve the following programme by use of the Kuhn-Tucker conditions:

Minimize $Z = x_1^2 + 5x_2^2 + 10x_3^2 - 4x_1x_2 + 6x_1x_3 - 12x_2x_3 - 2x_1 + 10x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \geq 4$

with all variables nonnegative. **(10 Marks)**

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